Electron spin transport through an Aharonov–Bohm ring—a spin switch

2004 J. Phys.: Condens. Matter 16 2043

(http://iopscience.iop.org/0953-8984/16/12/013)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **16** (2004) 2043–2052 PII: S0953-8984(04)71721-X

Electron spin transport through an Aharonov–Bohm ring—a spin switch

Cheng-Long Jia[1], **Shun-Jin Wang**[1,2], **Hong-Gang Luo**[1,3] **and Jun-Hong An**[1]

[1] Department of Modern Physics, Lanzhou University, Lanzhou 730000, People's Republic of China
[2] Department of Physics, Sichuan University, Chengdu 610064, People's Republic of China
[3] Institute of Theoretical Physics, Academia Scenical, Beijing 100080, People's Republic of China

Received 10 November 2003
Published 12 March 2004
Online at stacks.iop.org/JPhysCM/16/2043 (DOI: 10.1088/0953-8984/16/12/013)

Abstract

Electron spin transport through an Aharonov–Bohm ring driven by time-dependent inhomogeneous magnetic fields is treated. The system possesses an $su(2)_l \times su(2)_s$ dynamical symmetry in both orbital angular momentum space and spin space, and is thus proved to be integrable according to algebraic dynamics. Based on the analytical solutions, the relevant physical quantities such as electric current, spin current, magnetization and conductance are calculated. It is found that for a magnetic field with $\pi/2$ twist angle, the direction of spin-polarization will be reversed at zero magnetic flux. In the resonant rotating magnetic field, the spin transmission is oscillating with time t, and can reach unity, so that a complete spin flip can also be induced. The results obtained may be of practical significance for the design of nano-electromagnetic spin devices, such as a spin switch, in a controllable way.

1. Introduction

Much attention has been focused on spin related transport through a mesoscopic ring in the presence of an inhomogeneous magnetic field. For such a system, two problems are of great interest. One is the geometric phase. When a system evolves through an arbitrary path in its parameter space, the physical state acquires a memory of its motion in the form of dynamical phases and geometric phase, including the cyclic geometric phase [1] and its non-cyclic generalization [2]. The geometric phase of the wavefunction is a fascinating consequence of quantum mechanics and leads to various interference phenomena which are experimentally observable [3]. In recent experiments [4, 5], Yau *et al* observed evidence of a Berry phase acquired by a carrier of spin as it travels around a ring. On the other hand, due to the potential applications for quantum computing and spintronics, another interesting problem, which is the control of spin degree of freedom at the mesoscopic scale, has become the centre of attention.

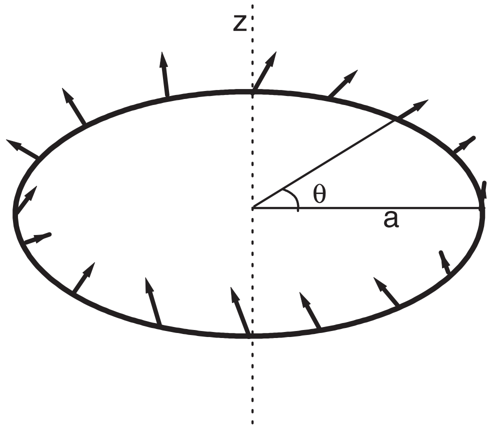

Figure 1. Overview of a mesoscopic ring system. The ring (thick circle) is embedded in an inhomogeneous time-dependent magnetic field (arrows) with tilt angle $\beta(\theta)$ and twist angle $\chi(\theta)$.

Frustaglia *et al* [6] studied a ballistic ring subject to an inhomogeneous circular (in-plane) magnetic field, and showed that the polarization direction of the transmitted spin-polarized electrons can be controlled via an additional perpendicular magnetic field such that spin flips are induced at half a flux quantum. Thus this system can be made as a device of spin switch and opens up the possibility of many proposed future applications, e.g., spin transistors [7], filters [8], and scalable devices for quantum information processing [9]. However, up to now, theoretical analysis [10–15] has been based on static magnetic fields and the corresponding results are deduced from autonomous systems, and, moreover, realistic experiments are carried out in time-dependence magnetic fields and the system becomes non-autonomous. The purpose of this paper is to treat this kind of non-autonomous quantum system and to study quantum transport of electrons in a mesoscopic ring embedded in a time-dependent inhomogeneous magnetic field. After the exact analytical solution of the system has been obtained by using an algebraic dynamical method [16], the properties of the system are calculated. We found that, by proper control of a rotating magnetic field, the spin flip can be manipulated at one's disposal. This gives us the possibility of designing nano-electromagnetic spin devices controlled by a time-dependent magnetic field.

2. Non-autonomous dynamics of a mesoscopic ring in time-dependent magnetic fields in general

Let us consider non-interacting electrons with effective mass m confined to a ring of radius a. The ring is embedded in a time-dependent inhomogeneous magnetic field $\mathbf{B}(t) = B_r(t)\hat{e}_r + B_\theta(t)\hat{e}_\theta + B_z(t)\hat{e}_z$, as depicted in figure 1. In cylindrical coordinates, the Hamiltonian for this system is taken to be,

$$\hat{H}(t) = \frac{1}{2ma^2}[\hat{P}_\theta - ea\mathbf{A}_\theta(t)/c]^2 + \mu\mathbf{B}(t)\cdot\hat{\boldsymbol{\sigma}}, \tag{1}$$

where $\hat{P}_\theta$ is the angular momentum operator conjugate to the coordinate θ, $\hbar\hat{\sigma}_i/2$ (with $i = r, \theta, z$) are the spin operators that satisfy $[\hat{\sigma}_i, \hat{\sigma}_j] = 2\mathrm{i}\epsilon_{ijk}\hat{\sigma}_k$, and μ is the magnetic moment. For the inhomogeneous textured magnetic field $\mathbf{B}(t) = \boldsymbol{\nabla}\times\mathbf{A}(t)$, the Zeeman term $\mu\mathbf{B}(t)\cdot\hat{\boldsymbol{\sigma}}$ couples the spin and orbital degrees of freedom and an effective spin–orbital coupling is thus resulted [11]. Assume that the in-plane magnetic field components $B_r(t)$

and $B_\theta(t)$ are all θ-independent but time-dependent, the system then possesses the cylindrical symmetry which leads to the conservation of total angular momentum $\hat{J}_z = \hat{P}_\theta + \frac{\hbar}{2}\hat{\sigma}_z$, namely $[\hat{J}_z, \hat{H}] = 0$. This makes us work in an invariant subspace labelled by a certain eigenvalue j_z of $\hat{J}_z$. In such an invariant subspace, the non-autonomous Hamiltonian becomes a linear function of the $\hat{\sigma}_i$,

$$\hat{H}(t) = \frac{\hbar\omega_0}{2}[j_z - \phi(t)]^2 + \frac{\hbar\omega_0}{8} + \frac{\hbar}{2}[\omega_r(t)\hat{\sigma}_r + \omega_\theta(t)\hat{\sigma}_\theta + \omega'_z(t)\hat{\sigma}_z], \tag{2}$$

where $\phi(t)$ is the enclosed magnetic flux in the unit of the flux quantum, and

$$\begin{aligned} &\omega_0 = \frac{\hbar}{ma^2}, \qquad \omega_r(t) = \frac{2\mu B_r(t)}{\hbar}, \qquad \omega_\theta(t) = \frac{2\mu B_\theta(t)}{\hbar}, \\ &\omega'_z(t) = \frac{2\mu B_z(t)}{\hbar} - \omega_0[j_z - \phi(t)]. \end{aligned} \tag{3}$$

One can see, comparing with other two components of the Larmor frequency ω_r and ω_θ, $\omega'_z(t)$ has been modified by the effective spin–orbital coupling. To solve the Schrödinger equation $\mathrm{i}\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}(t)|\Psi(t)\rangle$, let us perform a gauge transformation [16],

$$\hat{H} \to \hat{\overline{H}} = U_\mathrm{g}^{-1}\hat{H}U_\mathrm{g} - \mathrm{i}\hbar U_\mathrm{g}^{-1}\partial U_\mathrm{g}/\partial t, \tag{4a}$$

$$|\Psi(t)\rangle \to |\overline{\Psi}(t)\rangle = U_\mathrm{g}^{-1}|\Psi(t)\rangle, \tag{4b}$$

$$U_\mathrm{g}(t) = \exp[\mathrm{i}v_z(t)\hat{\sigma}_z]\exp[\mathrm{i}v_r(t)\hat{\sigma}_r] \tag{4c}$$

where the parameters $v_z(t)$ and $v_r(t)$ have definite physical meanings (see below). If the transformation parameters $v_r(t)$ and $v_z(t)$ are chosen in such a way that

$$\omega_r\cos 2v_z - \omega_\theta\sin 2v_z + 2\dot{v}_r = 0, \tag{5a}$$

$$\omega_r\cos 2v_r\sin 2v_z + \omega_\theta\cos 2v_r\cos 2v_z - \omega'_z\sin 2v_r - 2\dot{v}_z\sin 2v_r = 0, \tag{5b}$$

one has a special gauge transformation which diagonalizes the gauged Hamiltonian in the $\hat{\sigma}_z$ representation,

$$\hat{\overline{H}} = \frac{\hbar\omega_0}{2}[j_z - \phi(t)]^2 + \frac{\hbar\omega_0}{8} + \frac{\hbar}{2}\frac{\omega_r\sin 2v_z(t) + \omega_\theta\cos 2v_z(t)}{\sin 2v_r(t)}\hat{\sigma}_z. \tag{6}$$

Now the time-dependent dynamical symmetry of the Hamiltonian $\hat{H}(t)$ has been converted into the stationary symmetry of the gauged Hamiltonian $\hat{\overline{H}}$ by virtue of a proper choice of the gauge. Let $|m\rangle$ be the eigenstate of $\hat{\sigma}_z$ with eigenvalue $m(=\pm 1)$, the solution to the gauged Schrödinger equation $\mathrm{i}\hbar\frac{\partial}{\partial t}|\overline{\Psi}(t)\rangle = \hat{\overline{H}}|\overline{\Psi}(t)\rangle$ can be written down explicitly as

$$|\overline{\Psi}_{n,m}(\theta,t)\rangle = \mathrm{e}^{-\mathrm{i}\Theta_{n,m}(t)}\mathrm{e}^{\mathrm{i}n\theta}|m\rangle, \tag{7}$$

with $\Theta_{n,m}(t) = \int_0^t \overline{E}_{n,m}(t')\,\mathrm{d}t'$ and $\overline{E}_{n,m}(t) = \frac{\hbar\omega_0}{2}[(n-\phi)^2 + m(j_z - \phi)] + \frac{m\hbar}{2}\frac{\omega_r\sin 2v_z(t)+\omega_\theta\cos 2v_z(t)}{\sin 2v_r(t)}$. Here $n = j_z - \frac{m}{2}$ is the eigenvalue of the angular momentum operator $\hat{l}_z$ and is an integer number. Based on the gauge transformation (4), the solution to the original Schrödinger equation reads

$$|\Psi_{n,m}(\theta,t)\rangle = \mathrm{e}^{-\mathrm{i}\Theta_{n,m}(t)}\mathrm{e}^{\mathrm{i}n\theta}\sum_{m'} D^{1/2}_{m'm}(v_r)\mathrm{e}^{\mathrm{i}m'v_z}|m'\rangle, \tag{8}$$

where $D^{1/2}_{m'm}(v_r)$ is like the Wigner function defined as:

$$D^{1/2}(v_r) = \begin{bmatrix} \cos v_r(t) & \mathrm{i}\sin v_r(t)\mathrm{e}^{-\mathrm{i}\theta} \\ \mathrm{i}\sin v_r(t)\mathrm{e}^{\mathrm{i}\theta} & \cos v_r(t) \end{bmatrix}. \tag{9}$$

It is obviously that the bases $|\Psi_{n,m}(\theta, t)\rangle$ in each subspace labelled by j_z with different m are orthogonal. Furthermore, because j_z is a good quantum number and $\{\nu_r(t), \nu_z(t)\}$ are dependent on the quantum j_z, it can be proved that $|\Psi_{n,m}(\theta, t)\rangle$ are also orthogonal for different j_z. Therefore the bases $|\Psi_{n,m}(\theta, t)\rangle$ are complete and orthogonal in the whole Hilbert space. The general wavefunction of the system can thus be expanded in terms of them,

$$|\Psi(\theta, t)\rangle = \sum_{n,m} C_{n,m}|\Psi_{n,m}(\theta, t)\rangle = \sum_{n,m,m'} C_{n,m}\mathrm{e}^{-\mathrm{i}\Theta_{n,m}(t)}\mathrm{e}^{\mathrm{i}n\theta} D^{1/2}_{m'm}(\upsilon_r)\mathrm{e}^{\mathrm{i}m'\upsilon_z}|m'\rangle. \quad (10)$$

$C_{n,m}$ are time-independent expansion coefficients and completely determined by the initial condition. It is noted that when the magnetic field becomes static, this basis (8) is reduced to the result discussed in the previous paper [11, 12, 14].

Because the magnetic field is time-dependent, the energy of the non-autonomous system is not conserved,

$$\begin{aligned} E_{n,m}(t) &= \langle\Psi_{n,m}(t)|\hat{H}|\Psi_{n,m}(t)\rangle = \overline{E}_{n,m}(t) - m\hbar\dot{\upsilon}_z \cos 2\upsilon_r \\ &= \frac{\hbar\omega_0}{2}(n-\phi)^2 + \frac{m\hbar\omega_0}{2}\left(n + \frac{m}{2} - \phi\right)(1-\cos 2\upsilon_r) \\ &\quad + \frac{m\mu\hbar}{2}[\sin 2\upsilon_r(B_r \sin 2\upsilon_z + B_\theta \cos 2\upsilon_z) + B_z \cos 2\upsilon_r]. \end{aligned} \quad (11)$$

Here we can easily identify the first term as being the kinetic energy, the second term being the spin–orbit energy, and the last term being the Zeeman energy, respectively. Moreover, by using the basis $|\Psi_{n,m}(\theta, t)\rangle$, we can study other observable physical quantities of the system, such as the charge current $\langle\hat{J}^0\rangle$ and the spin current vector $\langle\hat{\mathbf{J}}\rangle$ [11],

$$\langle\hat{J}^0_{n,m}\rangle = n - eA_\theta/\hbar c + m[1-\cos 2\upsilon_r]/2, \quad (12)$$

$$\langle\hat{J}^z_{n,m}\rangle = m\cos 2\upsilon_r[n - eA_\theta/\hbar c + m(1-\cos 2\upsilon_r)/2] - \sin^2\upsilon_r/2. \quad (13)$$

with the other components $\langle\hat{J}^x_{n,m}\rangle$ and $\langle\hat{J}^y_{n,m}\rangle$ vanishing. Similarly, the magnetization vector $\hbar\langle\hat{\sigma}\rangle/2$ is given by

$$\langle\hat{\sigma}^r_{n,m}\rangle = m\sin 2\upsilon_r(t)\sin 2\upsilon_z(t), \quad (14a)$$

$$\langle\hat{\sigma}^\theta_{n,m}\rangle = m\sin 2\upsilon_r(t)\cos 2\upsilon_z(t), \quad (14b)$$

$$\langle\hat{\sigma}^z_{n,m}\rangle = m\cos 2\upsilon_r(t). \quad (14c)$$

The above equations indicate that the parameter $\upsilon_r(t)$ describes the deviation of the spin from the z-axis and $\upsilon_z(t)$ describes the spin rotation around the z-axis. From the expression $\langle\hat{J}^z_{n,m}\rangle = \langle\hat{J}^0_{n,m}\rangle\langle\hat{\sigma}^z_{n,m}\rangle - \frac{\sin^2\upsilon_r}{2}$, one can see that the term $-\frac{\sin^2\upsilon_r}{2}$ represents the coupling between $\hat{\sigma}^z$ and $\hat{P}_\theta - eA_\theta/c$, reflecting the quantum-mechanical correlation between the orbital angular momentum and spin, which is induced by the geometry of the inhomogeneous magnetic field.

Up to now, we have presented the exact solution of the non-autonomous mesoscopic ring system whose Hamiltonian is a function of $su(2)_l \times su(2)_s$ generators. This solution is quite general for any time-dependent magnetic field $\mathbf{B}(t)$. From the equations (5) with the initial conditions $\upsilon_r(0)$ and $\upsilon_z(0)$, the parameters $\upsilon_r(t)$ and $\upsilon_z(t)$ can be obtained, and all the properties of the 1D mesoscopic ring system can be calculated and in turn can be controlled by a proper setting up of the magnetic field $\mathbf{B}(t)$.

3. Special cases: a ring embedded in a rotating magnetic field

A simple and useful case is a magnetic field rotating around a z-axial at a fixed tilt angle and constant frequency,

$$B_r(t) = B_p \sin(2\omega t + \alpha), \tag{15a}$$

$$B_\theta(t) = B_p \cos(2\omega t + \alpha), \tag{15b}$$

$$B_z(t) = B_z, \tag{15c}$$

where $\chi = \frac{\pi}{2} - \alpha$ is the twist angle of the initial magnetic field. Intuitively, the spin will follow along with the rotating magnetic field, thus we can formally write down $v_z(t) = \omega t + \epsilon(t)/2$. Introducing this expression into equations (5), one has

$$2\dot{v}_r(t) + \omega_\mathrm{p} \sin(\alpha + \epsilon(t)) = 0, \tag{16a}$$

$$-\dot{\epsilon}(t) \sin 2v_r - (\omega_z' + 2\omega) \sin 2v_r + \omega_\mathrm{p} \cos 2v_r \cos(\alpha + \epsilon(t)) = 0. \tag{16b}$$

Due to the non-linearity of the above equations, it is generally difficult to obtain their analytical solutions. However, the numerical solution is feasible. Here we consider two limit cases: $\alpha \sim 0$ and $\alpha \sim \pi/2$. In the limit $\alpha \sim 0$, the initial configuration of the magnetic field is the same as that studied in [6, 12]; one can readily obtain the solutions of $v_r(t)$ and $v_z(t)$,

$$v_z(t) = \omega t, \qquad \tan v_r = \frac{\omega_\mathrm{p}}{\omega_z' + 2\omega}. \tag{17}$$

This shows that within the ring the electron travels with different spin tilt angle for distinct available Feynman paths labelled by j_z. With an increase of the rotating frequency ω, the spin tilt angle $2v_r$ decreases. When the magnetic field undergoes a periodic variation in the time interval $[0, \pi/\omega]$, the system returns to its initial state except for acquiring the total phase [17], $\Phi = -\Theta_{nm}(\pi/\omega) + \pi$, in which the non-adiabatic geometric phase, the Aharonov–Anandan phase, is $\Phi_\mathrm{AA} = \pi(1 - m \cos 2v_r)$. In the limit of strong spin–orbit coupling, the so-called adiabatic limit, the spin alignment follows the instant direction of the magnetic field, and $v_r \to \beta$ where $\beta = \arctan B_p/B_z$ is the tilt angle between the magnetic field $\mathbf{B}$ and the z-axis, the AA phase reduces to the adiabatic Berry phase $\Phi_\mathrm{B} = \pi(1 - m \cos \beta)$, and all carriers propagate with the same spin tilt angle β in the ring.

In the limit $\alpha \sim \pi/2$, equations (16) deduce a resonant solution,

$$v_z(t) = \omega t = \omega_z' t/2, \qquad v_r(t) = \omega_\mathrm{p} t/2. \tag{18}$$

In such a resonant configuration, the spin makes a spiral motion along the ring. After $v_r(t)$ evolves from 0 to $\pi/2$, electron completely reverses its spin direction, which implies that such a resonant magnetic field can be controlled to produce some nano-electro-magnetic spin devices, e.g., a spin switch. On the other hand, contrary to the case of $\alpha \sim 0$, after a cyclic evolution of the magnetic field and the Hamiltonian in the time interval $[0, \pi/\omega]$, the wavefunction $|\Psi_{n,m}(\theta, t)\rangle$, however, takes a non-cyclic evolution. Following the definition of the non-cyclic geometric phase in [2], one obtains the non-cyclic geometric phase as follows,

$$\Phi_\mathrm{g}(t) = -m\omega_z' t + m \frac{\omega_z'}{\omega_\mathrm{p}} \sin \omega_\mathrm{p} t, \tag{19}$$

which is similar to the result of Wagh and Rakhecha [18]. Using a polarized neutron beam, they determined this phase as well as interference amplitudes for non-cyclic spinor evolutions in a magnetic field.

4. Quantum transport of electrons through the ring

Now consider a ring coupled to two equivalent current leads. In this case, the ring is no longer closed, the leads break the rotational symmetry, and $\hat{J}_z$ is not conserved. The electrons propagating in the leads traverse the ring in both clockwise and counter-clockwise directions and the resulting transmission probability is determined by an interference. In the following, we shall focus our attention on the ballistic motion of electrons in the ring at the strong coupling limit [19]. We assume the two leads support only one open channel. At zero temperature, the spin-dependent conductance of such a mesoscopic system is given by the Landauer formula:

$$G = \frac{e^2}{h} \sum_{s,s'} T_{s's} = \frac{e^2}{2h} \sum_{s,s'} \left| \sum_i A^i_{s's} \right|^2. \tag{20}$$

The coefficients $A^i_{s's}$ denote the probability amplitude of the ith Feynman path from an incoming quantum channel with spin s to an outgoing channel with spin s'. Considering an incident electron with energy E_F and spin s in the right conductor. Depending on the spin alignment (m) and the direction of angular momentum (λ), the initial electronic state in the ring is a superposition of the four wavefunctions $|\Psi_{n^\lambda_m,m}(0,0)\rangle$, where the quantities n^λ_m are determined by the equation $E_F = E_{n,m}(0)$ in equation (11). In general, the energy of the incident electron, E_F, does not exactly coincide with the diabatic eigen energy in a closed ring. Therefore, there are four approximate integer solutions n^λ_m, which are positive n^+_+ and negative n^-_+ with $m = +1$, and positive n^+_- and negative n^-_- with $m = -1$, corresponding to four diabatic eigen wavefunctions.

As an incoming electron with arbitrary spin state $|s\rangle = C_\uparrow|\uparrow\rangle + C_\downarrow|\downarrow\rangle$ ($\sum_m C^2_m = 1$) enters the ring from the right lead, its initial spin state in the ring is $|\Psi_{n^\lambda_m,m}(0,0)\rangle = \sum_{m'} C_m D^{1/2}_{m'm}[v^r_{\lambda,m}(0), 0]|m'\rangle$. After time τ, the electron has propagated to the left lead along the four available Feynman paths, and corresponding wavefunctions are

$$|\Psi_{n^\lambda_m,m}(\pi,\tau)\rangle = \sum_{m'} C_m \mathrm{e}^{-\mathrm{i}\Theta_{n^\lambda_m,m}(\tau)} \mathrm{e}^{\mathrm{i}\lambda n^\lambda_m \pi} D^{1/2}_{m'm}(v^r_{\lambda,m}(\tau)) \mathrm{e}^{\mathrm{i}m' v^z_{\lambda,m}(\tau)} |m'\rangle. \tag{21}$$

These four waves interfere at $\theta = \pi$, and the transmission $T_{\kappa s}(\tau)$ ($\kappa = \uparrow, \downarrow$) is given as $T_{\kappa s}(\tau) = \frac{1}{2}|\sum_{\lambda,m}\langle\kappa|\Psi_{n^\lambda_m,m}(\pi,\tau)\rangle|^2$. In the resonant magnetic field ($\alpha \sim \pi/2$) or in the adiabatic limit for the case $\alpha \sim 0$, the four Feynman paths have the same spin tilt angle, $v^r_{\lambda,m}(t) \to v_r(t) = \omega_p t/2$ or $v^r_{\lambda,m}(t) \to v_r(t) = \beta$, respectively. And the transmissions are simple:

$$T_{\uparrow s}(\tau) = C^2_\uparrow \cos^2 v_r(\tau)[1 + \cos(\Delta\phi_{n^+_\uparrow - n^-_\uparrow})] + C^2_\downarrow \sin^2 v_r(\tau)[1 + \cos(\Delta\phi_{n^+_\downarrow - n^-_\downarrow})], \tag{22a}$$

$$T_{\downarrow s}(\tau) = C^2_\uparrow \sin^2 v_r(\tau)[1 + \cos(\Delta\phi_{n^+_\uparrow - n^-_\uparrow})] + C^2_\downarrow \cos^2 v_r(\tau)[1 + \cos(\Delta\phi_{n^+_\downarrow - n^-_\downarrow})], \tag{22b}$$

where $\Delta\phi$ is the phase difference contributed from the distinct interference Feynman path in the mesoscopic ring due to quantum coherence. By imposing equation $E_F = E_{n,m}(0)$ and after half an evolution, the phase difference between two spin waves with the same m travelling in opposite angular momentum direction can be calculated as,

$$\Delta\phi_{n^+_m - n^-_m} = -2\pi\phi - m\pi[1 - \cos 2v_r(0)].$$

One can identify the phase $-2\pi\phi$ as being the Aharonov–Bohm (AB) phase, and $-m\pi[1 - \cos 2v_r(0)]$ as being the Berry phase. It should be pointed out that, due to the spin precession in the time-dependent magnetic field, generally, the initial spin wave with opposite m and travelling in the same direction are not orthogonal, which is different from the result of Nitta *et al* [15].

For incoming *spin-unpolarized* currents, after summation over the spin indices κ and s, the total spin-dependent conductance of the mesoscopic system is,

$$G = \frac{e^2}{h}[1 + \cos 2\pi\phi \cos \pi(1 - \cos 2v_r(0))], \tag{23}$$

which exhibits Aharonov–Bohm oscillations with a period of $\phi_0 = hc/e$ [20]. However, because of the spin freedom, there is an additional initial phase contribution $\phi = \pi(1 -$

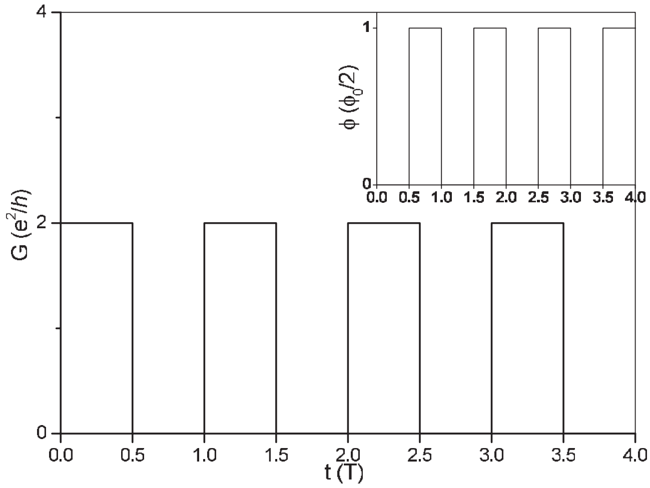

Figure 2. The total spin-dependent conductance for *z-polarized* electrons through a mesoscopic ring embedded in a rectangularly oscillating perpendicular magnetic field $B_z(t)$ (inset, $\phi = B_z(t)\pi a^2$) with period T. Such a system acts as a ideal diode.

$\cos 2v_r(0))$. If the initial spin state is *z-polarized*, i.e. $v_r(0) = 0$ or $\pi/2$, which can be satisfied in the two above mentioned magnetic fields, the conductance becomes zero, $G = 0$, at $\phi = \phi_0/2$. Under such conditions, the mesoscopic system is a ideal insulator. In other words, a mesoscopic ring embedded in a rectangularly oscillating perpendicular magnetic field $B_z(t)$ can be used as a diode (figure 2).

In the work of Frustaglia *et al* [6], they found that the polarization direction of initially polarized and transmitted electrons can be reversed at half a flux quantum. Motivated by their work, let us study the magnetoconductance of incoming spin-up polarized carriers, $|s\rangle = |\uparrow\rangle$ (equivalent results are obtained for spin-down incoming states). Firstly, we discuss the possibility of controlling spin polarization in the case of the magnetic field with zero α angle. From the discussion of the above section, we know that strong spin–orbit coupling (the adiabatic limit) leads to four available Feynman paths which have the same spin tilt angle, $v_r = \beta$. In the 'weak' magnetic field limit $B_z \to 0$, $B_p/B_z \to \infty$ and $\beta = \pi/2$, $\cos v_r$ goes to zero. In equations (22), we found that the transmission becomes: $T_{\uparrow\uparrow} \to 0$ and $T_{\downarrow\uparrow} \to 1$ (see in figure 3(a)). Hence, for zero flux, the transmitted carriers precisely reverse their spin-polarization and the ring acts as tunable spin-switches. This means that the spin flip can be realized experimentally as the ring only subject to an 'in-plane' magnetic field ($B_z = 0$), which is believed to be realistic. On the other hand, for the resonant magnetic field, because $v_r(t) = \omega_p t/2$, the transmission is oscillating with time t. We know that, for a 1D ring of radius a, the adiabatic separation of timescales implies that the Larmor frequency of spin precession, $\omega_s = 2\mu|B|/\hbar$, must be larger compared to the frequency $\omega_F = v_F/a$ of orbital motion with the Fermi velocity v_F around the ring [6, 12], i.e. $\omega_s/\omega_F \gg 1$. In a weak magnetic field $B_z \approx 0$, we have $\omega_p\tau/\pi \gg 1$. So, modulating the magnetic field B_p such that the time τ of electrons through the ring satisfies the condition: $\omega_p\tau/2 = k\pi + \frac{\pi}{2}$ (it is possible since $\omega_p\tau/\pi \gg 1$), the transmission $T_{\uparrow\uparrow}$ vanishes and only the $T_{\downarrow\uparrow}$ is left, and spin flip is also induced (see also figure 3(b)). Thus, the condition for spin flip in the resonant magnetic field again amounts to the 'weak' magnetic field limit. Hence, such a mesoscopic system can be used as a controller of spin current.

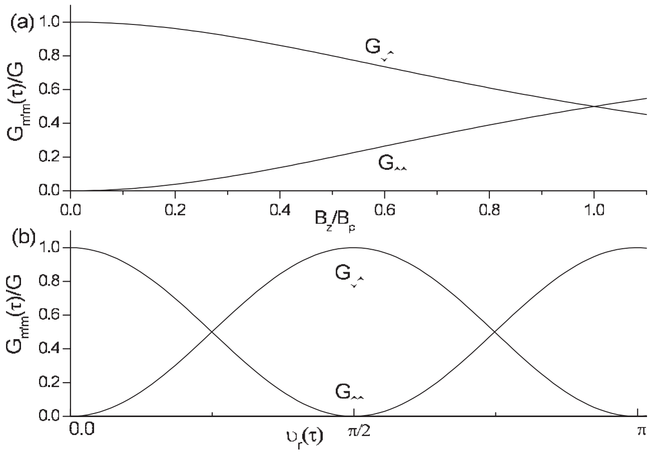

Figure 3. Conductance of spin-up polarized incoming carriers through a ring embedded in different initial configuration magnetic fields: (a) for the case of $\alpha \sim 0$, the spin-resolved conductance as a function of the ratio B_z/B_p; (b) for the resonant magnetic field ($\alpha \sim \pi/2$), the conductance oscillates with the spin tilt angle $\upsilon_r(\tau)$ at the left junction. Note the spin switching in the 'weak' B_z limit and at $\upsilon_r(\tau) = \pi/2$.

From the above investigation, we found that all the above phenomena, the spin flip, the spin current and the conductance of electron transport, etc, are intimately related to the coherence and interference of different Feynman paths. And the latter, the coherence and interference of the wavefunctions for different paths, in turn depend on the total phases acquired during their time evolution through the ring. The total phase of each wavefunction consists of three parts: the dynamical phase due to the dynamical evolution (the energy) of the wavefunction, the Aharonov–Bohm phase due to the effect of the magnetic flux, and the geometric phase due to the time variation of the parameters of the Hamiltonian. All the phases are controlled by the parameters (the time-dependent magnetic fields) of the Hamiltonian and they are different functions of the parameters. By a proper choice of the time-dependent magnetic fields, the desired phenomenon can be achieved.

So far we discussed the mesoscopic ring where the leads open only one channel. From expressions (22), we notice that the transmission is determined by two parameters, the spin tilt angle $2\upsilon_r$ and the phase difference $\Delta\phi$ contributed from the different Feynman paths. However, in the '*weak*' magnetic field limit, the spin tilt angle $\upsilon^r_{\lambda,m}$ is the same for available Feynman paths and the phase difference of the spin waves is the sum of the AB phase and the geometric (Berry) phase. Thus, the current modulation is reasonable even after averaging over energy between the first and the second open channel [6]. Furthermore, as discussion in [15], usually electron quantum interference devices have to be a single mode in order to obtain large modulation, because of the different phase shift which smears the interference effect. For multiple channels, modification of the above formulas is in order:

(1) the Landauer formulas for conductance should be generalized to the case of multiple channels as done in [21],
(2) the Hamiltonian of the system should be modified to include the effect of the extra leads (channels) (such as bias potentials of each leads along the circle), and
(3) the solutions of the Schrödinger equation should be obtained under the new boundary condition set by the extra leads.

All the above modifications constitute a complicated issue which should be investigated in future.

5. Conclusion and discussion

In conclusion, in the framework of algebraic dynamics, an Aharonov–Bohm ring embedded in a time-dependent inhomogeneous magnetic field has been investigated. We found that this non-autonomous quantum system possesses an $su(2)_l \times su(2)_s$ dynamical symmetry and is thus integrable. Employing the exact solutions, the electric current, spin current, magnetization and quantum transport driven by time-dependent magnetic fields have been calculated explicitly. For the periodically time-dependent magnetic field, the non-adiabatic cyclic and non-cyclic geometric phase are also computed. The most interesting results are obtained in the 'weak' magnetic field limit and the spin-orientation of the polarized carriers can be completely reversed. The obtained results may be of practical significance for the design of nano-electromagnetic spin devices in a controllable way, such as a spin switch and filter, scalable devices for quantum information processing, and so on.

Acknowledgments

This work was supported in part by the National Natural Science Foundation under the grant nos 10175029, 10375039 and 10004012, the Doctoral Education Fund of the Education Ministry and Post-doctoral Science Foundation, and the Nuclear Theory Research Fund of HIRFL of China.

References

[1] Berry M V 1984 *Proc. R. Soc.* A **392** 45
Aharonov Y and Anandan J 1987 *Phys. Rev. Lett.* **58** 1593
[2] Samuel J and Bhandari R 1988 *Phys. Rev. Lett.* **60** 2339
Pancharatnam S 1956 *Proc. Indian Acad. Sci.* A **44** 247
[3] Yang M J, Yang C H, Cheng K A and Lyanda-Geller Y B 2002 *Preprint* cond-mat/0208260
[4] Yau J B, De Poortere E P and Shayegan M 2002 *Phys. Rev. Lett.* **88** 146801
Anandan J 2002 *Science* **297** 1656
[5] Mal'shukov A G and Chao K A 2003 *Phys. Rev. Lett.* **90** 179701
Yau J B, De Poortere E P and Shayegan M 2003 *Phys. Rev. Lett.* **90** 179702
Wagh A G and Rakhecha V C 2003 *Phys. Rev. Lett.* **90** 119704
Yau J B, De Poortere E P and Shayegan M 2003 *Phys. Rev. Lett.* **90** 179704
[6] Frustaglia D, Hentschel M and Richter K 2001 *Phys. Rev. Lett.* **87** 256602
Frustaglia D and Richter K 2003 *Preprint* cond-mat/0309228
[7] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
[8] Gilbert M J and Bird J P 2000 *Appl. Phys. Lett.* **77** 1050
[9] Imamoglu A *et al* 1999 *Phys. Rev. Lett.* **83** 4204
[10] Meir Y, Gefen Y and Entin-Wohlman O 1989 *Phys. Rev. Lett.* **63** 798
[11] Loss D, Goldbart P M and Balatsky A V 1990 *Phys. Rev. Lett.* **65** 1665
Loss D and Goldbart P M 1992 *Phys. Rev.* B **45** 13544
Engel H-A and Loss D 2000 *Phys. Rev.* B **62** 10238
[12] Stern A 1992 *Phys. Rev. Lett.* **68** 1022
[13] Aronov A G and Lyanda-Geller Y B 1993 *Phys. Rev. Lett.* **70** 343
[14] Gao X C and Qian T Z 1993 *Phys. Rev.* B **47** 7128
Qian T Z and Su Z B 1994 *Phys. Rev. Lett.* **72** 2311
Qian T Z, Yi Y S and Su Z B 1997 *Phys. Rev.* B **55** 4065
[15] Nitta J, Meijer F E and Takayanagi H 1999 *Appl. Phys. Lett.* **75** 695
[16] Wang S J, Li F L and Weiguny A 1993 *Phys. Lett.* A **180** 189

Wang S J and Zuo W 1994 *Phys. Lett.* A **196** 13
Jie Q L, Wang S J and Wei L F 1997 *J. Phys. A: Math. Gen.* **30** 2077
[17] Landau L D and Lifshitz E M 1977 *Quantum Mechanics* 3rd edn (Oxford: Pergamon)
Moody J, Shapere A and Wilczek F 1986 *Phys. Rev. Lett.* **56** 893
[18] Wagh A and Rakhecha V C 1995 *Phys. Lett.* A **197** 107
Wagh A and Rakhecha V C 1995 *Phys. Lett.* A **197** 112
Wagh A and Rakhecha V C 1998 *Phys. Rev. Lett.* **81** 1992
[19] Büttiker M, Imry Y and Azbel M Ya 1984 *Phys. Rev.* A **30** 1982
[20] Gefen Y, Imry Y and Azbel M Ya 1984 *Phys. Rev. Lett.* **52** 129
[21] Büttiker M 1986 *Phys. Rev. Lett.* **57** 1761
Stone A D and Szafer A 1988 *IBM J. Res. Dev.* **32** 384
Baranger H U and Stone A D 1989 *Phys. Rev.* B **40** 8169
Shepard K 1989 *Phys. Rev.* B **40** 11623